A Batch Scheduling and Operator Assignment Model for Flow Shops to Minimize Total Actual Flow Time

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***Abstract*—This paper proposes a batch scheduling and operator assignment model where more than one operators can be assigned to each machine. There are a number of parts to be processed in batches where each batch is to be processed through a number of operations for which there are alternative operators with different setup and processing times. Each operator will be assigned to at most one machine, and at least one operator will be assigned to each machine. The decision variables are assignment of each operator to each machine, the number of batches, batch sizes and the schedule of the resulting batches. A proposed algorithm is developed by trying different number of batch, starting from one, and then increasing the starting number of batches one by one until the objective function value does not improve anymore. Numerical examples are demonstrated to show how the algorithm works and result global-optimal solutions for the investigated problem.**

***Keywords*—Batch scheduling, operator assignment, flow shop, actual flow time.**

I. Introduction

Scheduling is an activity to assign a number of resources (e.g. machines or operators) to processing a number of entities (e.g. tasks, jobs or batches) while minimizing a performance measure subject to a number of constraints [1]. Scheduling that considers machine as a resource has been recognized as a complex problem, so it is frequently studied separately with other resources, such as operator [2]. To represent the situation in industries better, scheduling models need to consider machine and operator simultaneously [3].

Scheduling that considers machine and operator simultaneously has been investigated in several papers, such as [4] which propose a machine and operator scheduling model considering operations sequence to minimize makespan in job shops, and [5] which propose a scheduling model that considers time lag between operations and due date to minimize personnel cost. In both papers, processing times are fixed.

In industries, processing time of an operation can have variations due to several causes, such as different skill levels of operators. This occurs in industries involving operators where there are alternative operators with different skill levels to perform the operation. Operator with a better skill will be able to perform the operation in a shorter time than others. This situation has been studied in [6] which propose a job scheduling model for single-stage systems with different operator setup times to minimize makespan. Reference [7] has extended the model proposed in [6] for flow shops and for manufacturing systems that perform production in batches, assuming that only one operator is assigned to each machine.

In addition to operator's skill, variations in processing time can also occur if the operation can be accelerated by allocating additional tools. This condition has been investigated in [8] and [9]. Additionally, process acceleration can be generated by assigning additional operator, e.g. in assembling work stations. The mathematical model for this acceleration has been studied in [10], [11] and [12] for different objective functions.

Some performance measures that frequently used in scheduling are makespan, flow time, lateness, tardiness and the number of tardy jobs [13]. These performance measures could not meet due date and reduce inventory level simultaneously. However, a performance measure so called actual flow time, defined in [14] as the interval between starting time of production and due date, can satisfy the need to meet due date and reduce inventory level simultaneously. Minimizing the actual flow time in scheduling means that parts do not have to arrive in the shop floor simultaneously at time zero, but they can arrive at the time their process is started, and the parts are delivered to the customer at their due date [14]. The batch scheduling and operator assignment model in [7] assumes that only one operator can be assigned to each machine, while in practical situation, assigning more than one operator to a machine may be possible. This paper extends the model proposed in [7] by allowing more than one operators assigned to each machine as studied in [10] and [11] with the objective to minimize total actual flow time as proposed in [14].

II. PROBLEM DEFINITION

Indices, parameters and variables used in this paper are shown as follows.

Indices:

m	=	index of machine, $m = 1, ..., c$,
o	=	index of operator, $o = 1, ..., b$,
i	=	index of batch position sequenced from the latest item due date, $i = 1, \ldots, N$.

Parameters:

c	=	number of machines,
b	=	number of operators ($b \geq c$),
d	=	due date of parts from time zero,
n	=	number of parts requested,
$s_{m,o}$	=	setup time per batch at machine m when performed by operator o,
$t_{m,o}$	=	processing time per part at machine m when performed by operator o,

Variables:

F	=	total actual flow time of all parts,
N	=	number of batches,
$Q_{[i]}$	=	batch size, the number of parts in batch i,
$B_{m,[i]}$	=	starting time of batch i at machine m,
$X_{m,o}$	=	binary variable that equals to 1 if operator o is assigned to machine m, equals to 0 if not,
S_m	=	setup time per batch at machine m,
T_m	=	processing time per part at machine m.

Let there be n parts to be processed in N batches, and each batch i will be processed through c machines in a flow shop. Each operation at machine m can be performed by one or more of b alternative operators, and each operator o performs the operation at machine m with setup times $s_{m,o}$ and processing times $t_{m,o}$. The decision variables in the model are assignment of operator o to machine m ($X_{m,o}$), the number of batch N, batch sizes $Q_{[i]}$, and the schedule of batch i at machine m ($B_{m,[i]}$). All operations must be finished no later than due date d. This study will develop a model of simultaneous batch scheduling and operator assignment to minimize actual flow time in flow shops. In this paper we assume that more than one operator can be assigned to perform operations at each machine, operator assignment is identical for all batches, batch sizes can be non-negative real number (not required to be integer) as used in [15], [16] and [17], and that setups are performed after the arrival of parts.

III. MODEL FORMULATION

This research extends the flow shop batch scheduling and operator assignment model in [7] by introducing the possibility to assign more than one operators to each machine as proposed in [10]. However, the mathematical model of the processing time developed in this paper is different with the model proposed in [10] in several things. First, the model in [10] is developed for a job scheduling problem, while in this paper, we develop a model for batch scheduling problem. Second, the model in [10] assume the processing time consists of constant and variable components, while in this paper, we assume a simpler model, where the processing time only has variable component. Last, while the model in [10] assume that operators have identical skill level, in this paper, we consider that operators have different skill level as proposed in [6].

Suppose that there are p operators with identical skill level, each of them is able to perform an operation in t time unit. If the operators are assigned to perform the operation together, we assume that the processing time will be t/p. Also suppose that there are q operators with different skill level, and each of them is able to perform an operation in $t_1, ..., t_q$ time unit. If we assign the operators to perform the operation, we assume that the processing time will be T, where $1/T = 1/t_1 + ... + 1/t_q$. This equation is used to calculate setup and processing times at each machine, S_m and T_m, as follows.

$$\frac{1}{S_m} = \sum_{o=1}^{b} \frac{1}{s_{m,o}}, \quad \forall m \tag{1}$$

$$\frac{1}{T_m} = \sum_{o=1}^{b} \frac{1}{t_{m,o}}, \quad \forall m \tag{2}$$

Batch scheduling and operator assignment problem to minimize actual flow time in flow shop production system can be formulated in model (3) to (16).

$$\min F = \sum_{i=1}^{N} \left(d - B_{1,[i]}\right) Q_{[i]} \tag{3}$$

subject to

$$B_{c,[i]} = d - \left(iS_c + T_c \sum_{z=1}^{i} Q_{[z]} \right), \quad \forall i \tag{4}$$

$$B_{m,[1]} = B_{m+1,[1]} - (S_m + T_m Q_{[1]}), \quad m < c \tag{5}$$

$$B_{m,[i]} \leq B_{m,[i-1]} - (S_m + T_m Q_{[i]}), \quad m < c,\ i > 1 \tag{6}$$

$$B_{m,[i]} \leq B_{m+1,[i]} - (S_m + T_m Q_{[i]}), \quad m < c,\ i > 1 \tag{7}$$

$$B_{1,[N]} \geq 0 \tag{8}$$

$$\frac{1}{S_m} = \sum_{o=1}^{b} \frac{X_{m,o}}{s_{m,o}}, \quad \forall m \tag{9}$$

$$\frac{1}{T_m} = \sum_{o=1}^{b} \frac{X_{m,o}}{t_{m,o}}, \quad \forall m \tag{10}$$

$$\sum_{i=1}^{N} Q_{[i]} = n \tag{11}$$

$$\sum_{o=1}^{b} X_{m,o} \geq 1, \quad \forall m \tag{12}$$

$$\sum_{m=1}^{c} X_{m,o} \leq 1, \quad \forall o \tag{13}$$

$$X_{m,o} \in \{0,1\}, \quad \forall o, \forall i \tag{14}$$

$$1 \leq N \leq n \tag{15}$$

$$Q_{[i]} > 0, \quad \forall i \tag{16}$$

Objective function (3) is the minimization of total actual flow time, i.e. the sum of batch actual flow time multiplied by its size. Batch actual flow time is the time spent by the batch on the shop floor, i.e. the time length from the starting of the batch process at machine 1 to the item's due date. The starting time of each batch at each machine is determined in constraint (4) to (7). Since the model aims to minimize actual flow time, the scheduling is performed backwards, and the batch starting times at the last machine (machine c) are calculated first. Constraint (4) states that the starting time of batch i at machine c is the due date less by the cumulative setup and processing times from batch 1 to batch i. Constraint (5) states that the starting time of batch 1 at machine m is calculated backward from machine c to machine m. Constraint (6) and (7) state that the starting time of batches other than batch 1 at other than machine c is the earliest between the starting time of the previous batch at the same machine and the starting time of the batch on the next machine, less by its setup and processing time. Constraint (8) states that the starting time of the earliest batch processing must not be before time zero. Constraint (9) and (10), as taken from equation (1) and (2), calculate setup and processing time at machine m based on operator assignment to machine m. Constraint (11) states that the number of parts in all batches must be equal to the number of parts requested. Constraint (12) states that at least one operator is assigned to each machine; while constraint (13) states that each operator can only be assigned to maximum one machine. Constraint (14) states that the operator assignment variables are binary numbers, constraint (15) states that the number of batches is from one to the number of parts, and constraint (16) states that batch sizes must be positive.

IV. ALGORITHM

The problem formulated in (3) to (16) is not linear and not convex because the value of N is not given and it contains discrete variables $X_{m,o}$. Therefore, the problem will be solved by first setting the batch number, N. An algorithm is developed to solve the problem by trying several N values, starting from one, and increasing N one-by-one until the objection function value does not improve anymore. The solution method for the problem is described in the following algorithm.

Algorithm.

Step 1. Set parameters c, b, n, $s_{m,o}$, $t_{m,o}$, and d. Go to Step 2.
Step 2. Set $N = 1$. Go to Step 3.
Step 3. Solve the model formulated in (3)–(16), determine the best F at the current N. Go to Step 4.
Step 4. If $F < F^*$ or F^* has not been set, then set F as F^*, the current best solution, and go to Step 5. If not, go to Step 6.
Step 5. If $N = n$, go to Step 6. If not, set $N = N + 1$, go to Step 3.
Step 6. Stop. The current F^* is the optimal solution for the problem.

V. NUMERICAL EXAMPLES

The proposed model and algorithm developed in this paper are applied to several examples with different values of parameters, and in this section, we show one of them. In the example, we set the parameters $c = 3$, $b = 5$, $n = 50$, $d = 2000$, $s_{m,o}$ = (52 85 60 70 90, 81 42 69 58 56, 64 55 41 81 74) and $t_{m,o}$ = (9 8 5 12 13, 7 10 10 7 13, 14 13 9 9 7). The algorithm is run by using the Lingo 12.0 software installed in a Core i5-6200U with 8 GB RAM computer. The computation for the example is shown in Table 1, while the optimal solution is shown in Table 2. The optimal schedule for the example is presented in a Gantt-chart shown in Fig. 1, and the step-by-step computation of the algorithm for the example is shown as follows.

Step 1. The parameter values of the example are set. Go to Step 2.
Step 2. We set $N = 1$. Go to Step 3.
Step 3. Solving the model in (3)–(16) with $N = 1$, we get $F = 38892.3$, then go to Step 4.
Step 4. Since F^* has not been set, we set F as F^*. Go to Step 5.
Step 5. Since $N = 1$ has not reached $n = 50$, we set $N = N + 1 = 2$ and back to Step 3.
Step 3. Solving the model in (3)–(16) with $N = 2$, we get $F = 26343.3$, then go to Step 4.
Step 4. Since $F < F^*$, we set F as F^*. Go to Step 5.
Step 5. Since $N = 2$ has not reached $n = 50$, we set $N = N + 1 = 3$ and back to Step 3.
Step 3. Solving the model in (3)–(16) with $N = 3$, we get $F = 23192.3$, then go to Step 4.
Step 4. Since $F < F^*$, we set F as F^*. Go to Step 5.
Step 5. Since $N = 3$ has not reached $n = 50$, we set $N = N + 1 = 4$ and back to Step 3.
Step 3. Solving the model in (3)–(16) with $N = 4$, we get $F = 22540.7$, then go to Step 4.
Step 4. Since $F < F^*$, we set F as F^*. Go to Step 5.
Step 5. Since $N = 4$ has not reached $n = 50$, we set $N = N + 1 = 5$ and back to Step 3.
Step 3. Solving the model in (3)–(16) with $N = 5$, we get $F = 22532.9$, then go to Step 4.
Step 4. Since $F < F^*$, we set F as F^*. Go to Step 5.
Step 5. Since $N = 5$ has not reached $n = 50$, we set $N = N + 1 = 6$ and back to Step 3.
Step 3. Solving the model in (3)–(16) with $N = 6$, we get $F = 22538.1$, then go to Step 4.
Step 4. The current F (22538.1) is not lower than the current F^* (22532.9) obtained at $N = 5$, so we go to Step 6.
Step 6. We stop the computation. The current F^* is the optimal solution.

TABLE 1. COMPUTATION FOR THE EXAMPLE

N	1	2	3	4	5	6
Operator assignment#	3.14.25	3.12.45	3.14.25	3.14.25	3.14.25	3.14.25
F	38892.3	26343.3	23192.3	22540.7	22532.9*	22538.1

Note: # Assignment 3.14.25 means that operator 3 is assigned to machine 1, operator 1 and 4 are assigned to machine 2, operator 2 and 5 are assigned to machine 3.
* The best solution.

TABLE 2. THE OPTIMAL SOLUTION FOR THE EXAMPLE

	Batch i	1	2	3	4	5
	$Q_{[i]}$	7.6	11.5	15.7	13.6	1.6
Schedule	Machine 1	1775.7	1658.0	1519.4	1391.5	1323.6
	Machine 2	1873.6	1775.7	1658.0	1572.1	1532.8
	Machine 3	1934.0	1849.9	1746.8	1653.5	1614.7

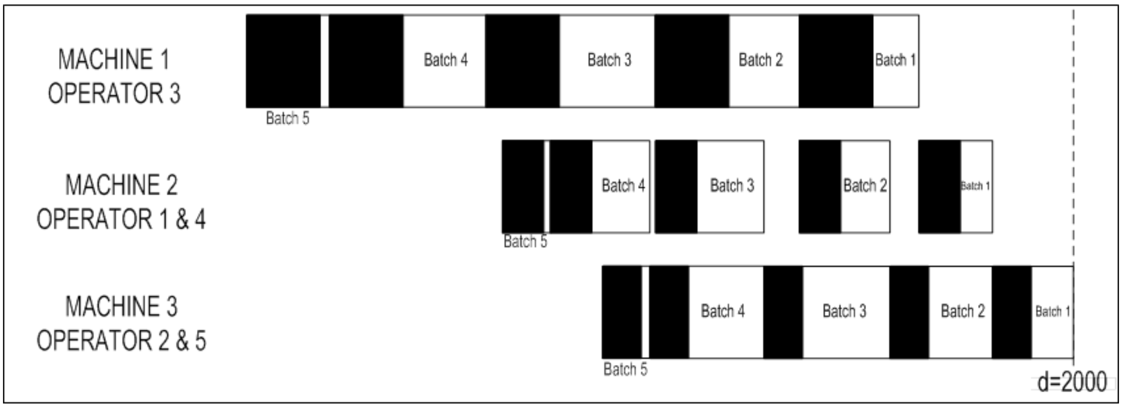

Fig. 1. Gantt chart for the optimal solution

From the numerical example we can analyze that in order to find the optimal solution, we need to repeat Step 3 to 5 in the proposed algorithm several times with different N values, until the objective function stop improving. More batches in the optimal solution require more repetition in the computation and require a longer computation time.

From the optimal solution of the example we can discuss several things. Since the model proposed in this paper is developed from the model in [7], it is noteworthy to compare the solution resulted by both models from a single data set. Solving the example presented in this section using the model and the algorithm developed in [7], we get an optimal solution: N = 6, F = 32532.6, and the operator assignment is 3.4.5. The model in this paper results a 30.7 percent better total actual flow time than the model in [7] for the example, since it allows a process acceleration by assigning additional operator. However, this model does not determine whether the total actual flow time improvement is worth for the cost of assigning additional operators.

The operator assignments resulted from both models are also necessary to discuss. In the optimal solution, the model in this paper utilizes all available operators (with assignment 3.14.25), while the model in [7] uses only operator three operators (with assignment 3.4.5). These assignments can be compared to the assignment resulted by the Hungarian method [18], which can be determined by firstly calculating $\tau_{m,o} = s_{m,o} + n.t_{m,o}$ for all m and o, using the example parameters, and then using $\tau_{m,o}$ values as parameters for the Hungarian method calculation (m becomes row, o becomes column). As the number of operators is greater than the number of machines in the example, we add dummy machines and set M (a big number) as the $\tau_{m,o}$ values of the dummy machines. The operator assignment resulting from the Hungarian method (omitting dummy machines) is 3.4.5, as shown in Table 3, which is identical with the result of [7] and partially identical with the result of the proposed model. This shows that operator 3, 4 and 5 have better skill so they are prioritized for machine 1 to 3, while operator 1 and 2 are additionally assigned to machine 2 and 3.

TABLE 3. THE HUNGARIAN METHOD FOR THE EXAMPLE

$\tau_{m,o}$		Operator o				
		1	2	3	4	5
Machine m	1	502	485	310*	670	740
	2	431	542	569	408*	706
	3	764	705	491	531	424*
	4	M*	M	M	M	M
	5	M	M*	M	M	M

Note: # $\tau_{m,o}$ is the total time required by operator o to perform operation at machine m.
* The selected assignment.

We need to remind that parts arrive to the shop floor at different times, i.e. at the beginning of the batch processing at machine 1. After processing, parts in a batch are delivered to the customer at the due date. This means that parts in batches with higher i have longer actual flow time, so the batch sizes tend to (but not always) decrease from batch 1 to batch N. This behaviour appears in all data sets we try.

Last, the proposed algorithm stops increasing the number of batch when the objective function value stops improving. The solution still may improve at a higher number of batches. However, in the numerical example shown, the objective function value does not improve when the number of batch is increased, so the resulting solution is global optimal.

Computation time required to find a solution tends to increase when N increases. This occurs because when N increases, the number of $Q_{[i]}$ and $B_{m,[i]}$ variables involved in the model also increases. This makes the number of possible solutions increases, thus it takes more time to search the best solution. For the given example, the computation time is no more than 1 second when $N = 1$, and approximately 4 minutes when $N = 6$. These computation times are reasonable; however, for bigger size cases, the computation time may be longer. Additionally, the computation time is also affected by the other parameter values, such as number of parts, set up and process times.

The proposed algorithm in this paper does not guarantee a global optimal solution, as it stops the solution searching when the objective function value stops improving. To find a global optimal solution convincingly, all possible N values must be tried; the algorithm should only stop when $N = n$. However, all data sets we solve using the proposed algorithm result global optimal solutions, which indicates that the algorithm is a reasonable trade-off to get a good solution in a reasonable computation time.

The model developed in this study can be applied for flow shop systems with any number of machines that have alternative operators in the process, and each operator can only be assigned to maximum one machine. The number of operators must be at least equal to the number of machines. In addition, this model can only be used if the setup and process of an operation is performed by the same operator.

VI. Concluding Remarks

This study developed a batch scheduling and operator assignment model to minimize actual flow time in flow shop production systems where a process acceleration is possible through additional operator assignment. The model developed in this study can be used for all flow shop systems where there are alternative operators for each operation, and each operator can only be assigned to maximum one machine. The algorithm works by increasing the number of batches, starting from one, until the number of batch where the solution does not improve anymore. The numerical examples examined in this paper result global optimal solutions. A future work of this paper needs to consider learning and forgetting processing experienced by operators, thus making the operators possible to experience joint-learning and forgetting processes.

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