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A Record-to-Record Travel Algorithm for Multi-product and Multi-period Inventory Routing Problem

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***Abstract*— The Inventory Routing Problem (IRP) is a problem that can be found in distributing goods. It considers the two important aspect such as inventory management and vehicle routing processes. This paper addresses the IRP multi-period, multi-product, and multi-supplier with capacitated homogenous fleet that housed at depot. Products are transport from supplier to assembly plant in finite horizon. The objective of this study is to minimize inventory and travel cost in distribution processes. A two stage algorithm is proposed. In the first stage, the initial solution is obtained using the least cost insertion algorithm. The results are improved in the second stage with a record-to-record travel algorithm that employ the 1-0 and 2-Opt local search. The algorithm is tested using the data sets from the literature and the data is processed using the software program that has been built. The results obtained are good compare to previous research, and the proposed algorithm produce short computational time.**

Keywords-inventory routing problem; multi-period, a record-to-record; logistic distribution

I. Introduction

Inventory management and transportation are two important aspects in the Supply Chain Management (SCM) and both have a role in achieving product availability, consistency of deliveries, accuracy in inventory or demand management, and improving effectiveness of the distribution process. The integration of inventory management and transportation, known as the inventory routing problem (IRP) which has a purpose for make distribution strategy in minimizing the total cost of vehicle routing and inventory management [1] [2] [3]. IRP can be categorized based on several aspects, such as the number and type of fleet (homogenous or heterogenous), number of periods, product, supplier, discrete or continuous time, demand may be deterministic or stochastic, planning horizon, and it can be applied in various distribution systems, such as maritime, automotive, or supermarket industries [4] [5].

Recently, many researchers have solved the problem in the IRP area with many approaches, for example, Abdelmaguid, Dessouky, and Ordonez [6] study IRP multi-period for a set of customers who receive units of a single item from a depot with infinite supply. Heuristic approaches are used to solve the IRP multi-period and the result of the improvement heuristics approach can approximate solution and demonstrate effectiveness through computational experiments. Similar with Abdelmaguid, Dessouky, and Ordonez [6], Bard & Nananukul [7] also study about the IRP multi-period in production systems. In this research, IRP is formulated as a mixed integer programming with the objective of maximizing the net benefits associated with making deliveries in a specific time period to a widely dispersed set of customers.

Archetti, Betazzi, Hertz, and Speranza [4] consider an IRP in discrete time where a supplier has to serve a set of customers over a multiperiod horizon. Archetti, Betazzi, Hertz, and Speranza [4] proposed a model that can minimize the sum of inventory and transportation costs. This study combines a tabu-search scheme with ad-hoc designed mixed-integer programming models. This study proved that heuristic can be used effectively in IRP multi-period case. A genetic algorithm is used by Hiassat, Diabat, and Rahwan [8] that address location-inventory-routing problem model for perishable products. The model determines the number and location of required warehouses, the inventory level at each retailer. Moin, Salhi, and Aziz [1] also solve the problem in a many-to-one distribution network consisting of an assembly plant and many distinct suppliers with genetic algorithm. This research considered a finite horizon where a fleet of capacitated homogenous vehicles, housed at depot, transport products from the suppliers to meet the demand specified by assembly plant in each period. The result of this research is several medium and small sized problems are solved with reasonable time. Beside that, many researcher use other heuristic approach to solve IRP case [9] [10].

Based on previous research that have been described, they approach are not guaranteed to produce an optimal solution and the runtime of the computational test is not always efficient. In vehicle routing problem (VRP) area that is a variant of IRP, Li, Golden, and Wasil [11] used record-to-record travel algorithm for solving the problem. The case of this research is in the heterogenous fleet VRP with several different types of vehicle can be used to service the customer. Li, Golden, and Wasil [11] have done testing to compare several approaches for solving the VRP such as heuristic approach, threshold accepting, and record-to-record travel algorithm. The result of this approach comparison, record-to-record travel algorithm produced new best-known solutions and was reasonably fast. A record-to-record travel algorithm is a deterministic variant of simulated annealing that can do the uphill moves in order to avoid becoming trapped at a poor local minimum [12]. Therefore, in this research a record-to-record travel algorithm is used to solve an IRP multi period case to search a best solution in short computational time. Moreover, the objective of this research is to solve the problem with multi-product, multi-period, multi-supplier, where a fleet capacitated homogenous vehicle, housed at a depot, and transport product from the supplier to assembly plant in finite horizon time same as the problem in Moin, Salhi, and Aziz [1]. In addition, this research explicitly modeled at various additional constraints, such as length limit of fleet, and maximum number of stop with record-to-record travel to get the best solution in short computational time.

II. Problem Description

In this research, we consider a similar case with Moin, Salhi, and Aziz [1] that consist of multi-product, multi-period, multi-supplier, where a fleet capacitated homogenous vehicle, housed at a depot, and transport product from the supplier to assembly plant in finite horizon. In addition, in this research, there are a fleet length limit and maximum number of stop constraint. The Network consists of a one depot, M supplier with distinct product, and one assembly plant. The vehicle returns to depot after transport the product from supplier to assembly plant. This model assumed the product always ready to be sent from supplier to assembly plant. There is no shortage, backordering/backlogging in this model. Inventory costs will arise when there are products stored on the assembly plant and exceed the demand in that period. The demand at the assembly plant is deterministic and the number of capacitated homogenous fleet is unlimited. The transportation cost per trip consists of fixed cost and variable cost for travel distance. The objective of this research is to minimize the total transportation cost and inventory cost over the planning horizon using a record-to-record travel algorithm.

III. Mathematical Formulation

The following are the mathematical formulation based on Moin et al. (2011):

Indices

$\tau = \{1, 2, \dots, T\}$ period

$S = \{1, 2, \dots, M\}$ a set of suppliers that suppliers product i where supplier $i (i \in S)$

$D = \{0\}$ depot

$A = \{M + 1\}$ assembly plant

Parameter

C vehicle capacity

F fixed vehicle cost per trip

V travel cost per unit distance

L length limit of vehicle for each trip

B maximum vehicle number of stop for each trip

r_{ij} travel distance from supplier i to j where $r_{ij} = r_{ji}$ and $r_{ik} + r_{kj} \geq r_{ij}$ (the triangle inequality), for any i, j and k with $i \neq j, k \neq i$ and $k \neq j$

h_i inventory holding cost for product i supplier i per period in assembly plant

inv_{i0} beginning inventory level (period 1) of product i supplier i at the assembly plant

d_{it} demand from supplier i product i at the assembly plant in period t

Variables

x_{ijt} number of times visited *(from i to j)* by vehicles in period t

a_{it} total amount to be picked-up at supplier i in period t

inv_{it} inventory level of product i supplier i at the assembly plant at the end of period t

q_{ijt} quantity transported (from i *to j)* in period t

The mathematical formulation for IRP is given as follows:

$$Y = min \ \ V\left(\sum_{\substack{j\in S\\ j\neq i}}\sum_{i\in S\cup D} r_{ij}\left(\sum_{t\in\tau} x_{ijt}\right) + \sum_{i\in S} r_{i,M+1}\left(\sum_{t\in\tau} x_{i,M+1,t}\right)\right) + r_{M+1,0}\sum_{t\in\tau}\sum_{i\in S} x_{0it} + F\sum_{t\in\tau}\sum_{i\in S} x_{0it} + \sum_{i\in S} h_i\left(\sum_{t\in\tau} inv_{it}\right) \tag{1}$$

Subject to

$$inv_{it} = inv_{i,t-1} + a_{it} - d_{it}, \ \ \forall i \in S, \forall t \in \tau \tag{2}$$

$$\sum_{\substack{i\in S\cup D\\ i\neq j}} q_{ijt} + a_{jt} = \sum_{\substack{i\in S\cup A\\ i\neq j}} q_{jit}, \ \ \forall j \in S, \forall t \in \tau \tag{3}$$

$$\sum_{i\in S} q_{i,M+1,t} = \sum_{i\in S} a_{it}, \ \ \forall t \in \tau \tag{4}$$

$$\sum_{\substack{i\in S\cup D\\ i\neq j}} x_{ijt} = \sum_{\substack{i\in S\cup A\\ i\neq j}} x_{jit}, \ \ \forall j \in S, \forall t \in \tau \tag{5}$$

$$\sum_{j\in S} x_{ijt} = \sum_{j\in S} x_{jkt}, \ \ i \in D, k \in A, \forall t \in \tau \tag{6}$$

$$q_{ijt} \leq C x_{ijt}, \ \ \forall i \in S, \forall j \in S \cup A, i \neq j, \forall t \in \tau \tag{7}$$

$$\sum_{\substack{j\in S\\ j\neq i}}\sum_{i\in S\cup D} r_{ij}\left(\sum_{t\in\tau} x_{ijt}\right) + \sum_{i\in S} r_{i,M+1}\left(\sum_{t\in\tau} x_{i,M+1,t}\right) + r_{M+1,0}\sum_{t\in\tau}\sum_{i\in S} x_{0it} \leq L, \ \ \forall i \in S, \forall j \in S \cup A, i \neq j, \forall t \in \tau \tag{8}$$

$$x_{ijt} \leq B, \ \ \forall i \in S, \forall j \in S \cup A, i \neq j, \forall t \in \tau \tag{9}$$

$$inv_{it} \geq 0, \ \ \forall i \in S, \forall t \in \tau \tag{10}$$

$$q_{it} \geq 0, \ \ \forall i \in S, \forall t \in \tau \tag{11}$$

$$x_{ijt} \in \{0,1\}, \ \ \forall i, j \in S, \forall t \in \tau \tag{12}$$

$$x_{0jt} \geq 0 \ and \ integer, \ \ \forall j \in S, \forall t \in \tau \tag{13}$$

$$x_{i,M+1,t} \geq 0 \ and \ integer, \ \ \forall i \in S, \forall t \in \tau \tag{14}$$

$$x_{ijt} = 0, \ \ i \in D, j \cup A, \forall t \in \tau \tag{15}$$

$$x_{ijt} = 0, \ \ i \in S, j \cup D, \forall t \in \tau \tag{16}$$

$$x_{ijt} = 0, \ \ i \in A, j \cup S, \forall t \in \tau \tag{17}$$

$$q_{ijt} \geq 0, \ \ \forall i \in S, \forall j \in S \cup A, \forall t \in \tau \tag{18}$$

$$q_{0it} = 0,\ \ \forall i \in S, \forall t \in \tau \tag{19}$$

The explanation of the mathematical formulations are as follows: (1) The objective function that consist of travel cost, fixed cost, and inventory cost; (2) Balance equation of inventory at the assembly plant for each product and each period; (3) Balance flow at each supplier; (4) The accumulative amount of picked-up quantities at the assembly plant; (5 and 6) Constraint for balance number of arrival vehicle and departure vehicle from depot to supplier, and assembly plant; (7) Vehicle capacity for each trip in all period; (8) Length limit of fleet for each trip in all period; (9) Maximum fleet number of stop for each trip in all period; and the remaining constraints are the nonnegativity constraints.

IV. A Record-to-Record Travel Algorithm in IRP

```
M = 120; K = 20, I = 60,
Global Record = ∞, Global Deviation =  0.01 x Global Record
Generate a feasible least travel cost insertion algorithm
Set Record = the objective function value of current solution
Set Deviation = 0.01 x Record
Set iteration = 0
Do while iteration ≤ M
    Set count = 0
    Do while count ≤ K
        For i = 1 to I
            One-point-move (within and between route) with a
record-to-
            record travel algorithm, Two-opt-move within route
            If Solution Status = “No Feasible” Then
                Go To Line of Local Search
            End If
            If Solution Status = “Feasible” Then
                If Current Solution < Record + Deviation Then
                    Set Record = Current Solution
                    Set Deviation = 0.01 x Record
                    Set count = 0
                End If
            End If
        Next i

Line of Local Search:

        One-point-move (within and between route), Two-opt-
move within
        route. Only downhill moves are allowed.
        If Solution Status = “Feasible” Then
            If Current Solution < Record + Deviation Then
                Set Record = Current Solution
                Set Deviation = 0.01 x Record
                Set count = 0
            End If
        End If
        count = count + 1
    Loop
    If Record < Global Record + Global Deviation
        Set Global Record = Record
        Set Global Deviation = 0.01 x Global Record
        Set iteration = 0
    End If
    Iteration = iteration + 1
Loop
Output solution of Global Record
```

Figure 1. A record-to-record travel algorithm in IRP.

The record-to-record travel algorithm is a deterministic variant of simulated annealing that can do the uphill moves in order to avoid becoming trapped at a poor local minimum [9]. Let *A* as the current solution and *A'* as an alternative solution. Record is the best solution observed so far and the record deviation is B% of Record. If the result of *A'* is less than Record + Deviation, then *A'* is the new solution. Here, a record-to-record travel algorithm from Li, Golden, and Wasil [9] is adapted to be implemented in the inventory routing problem. The record-to-record travel algorithm for the inventory routing problem for this research can be seen in Figure 1.

In Figure 1, I = represent as number of iterations record-to-record travel with uphill moves runs on the current solutions; K is the maximum number of iterations allowed when the local record has not been updated; and M is the maximum number of iterations allowed when the global record has not been updated. Based on our experiment, the best result is given if we use the value of M, K, I are 120, 20, 60 respectively.

V. RESULT AND DISCUSSION

The proposed method is coded in Visual Studio 2017 and run using a computer with i3 with 4 GB of RAM. This software is built in user interface of trigger input and reporting the best solution of the experiment. An example of code editor a record-to-record travel and user interface is given by Figure 2. In this research, data sets are used to test the proposed algorithm are obtained from Moin et al. (2011). There are 4 data sets that are S12T5 (12 supplier, 5 periods), S12T10, S20T5, and S20T10. Besides that, in this study the data set was used for each 5 period also. For each data set there is the number of record-to-record parameter such as the value of C, F, V, L, B, and then there are coordinate (x,y) and holding cost for each supplier, coordinate of depot and assembly plant, planning horizon, and demand for multi-period. The characteristic parameter of the record-to-record travel algorithm can be seen in Table I.

TABLE I. PARAMETER OF RECORD-TO-RECORD TRAVEL ALGORITHM

Data Sets	S12T05	S12T10	S20T05	S20T10
C	10	10	10	10
F	20	20	20	20
V	1	1	1	1
L	140	140	140	140
B	10	10	10	10
Range of holding costs	[3, 27]	[3, 27]	[3, 27]	[3, 27]
Range of demand	[1, 4]	[1, 4]	[1, 4]	[1, 4]
Coordinate of depot	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Coordinate of depot	(10, 20)	(10, 20)	(10, 20)	(10, 20)

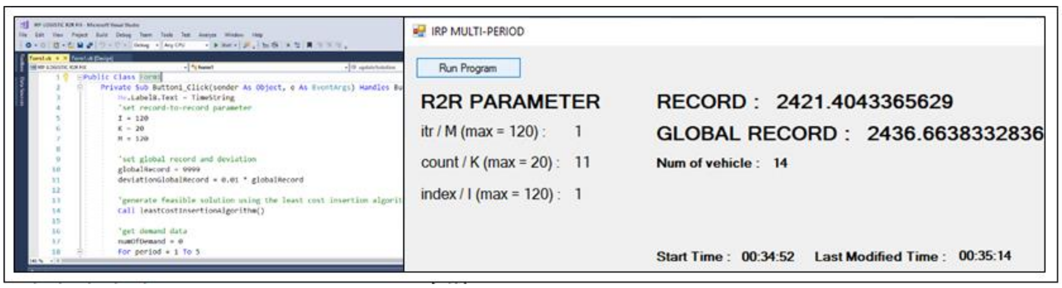

Figure 2. Example of code editor and user interface software program

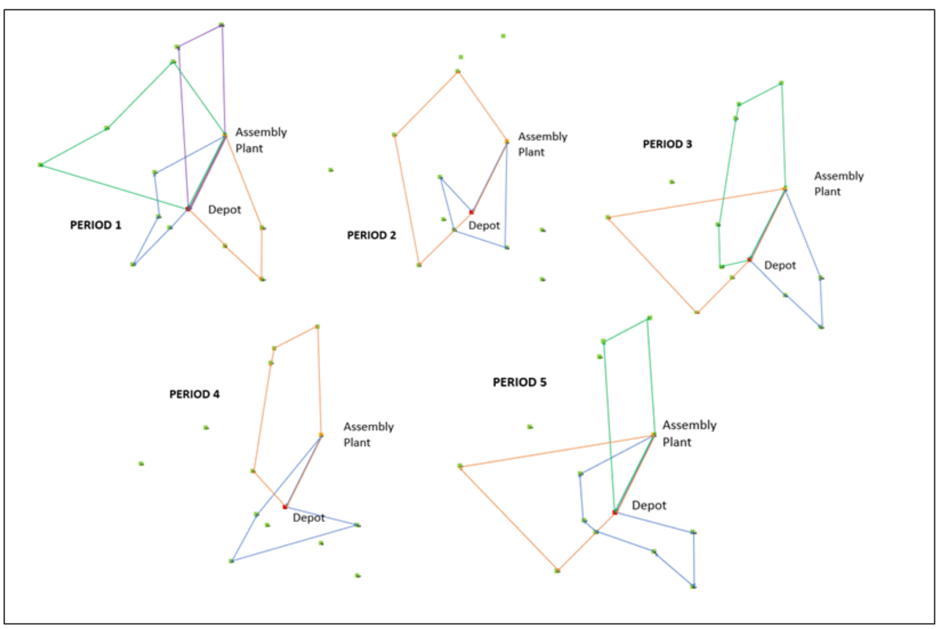

Figure 3. The route result of S12T05 with a record-to-record algorithm

The result obtained and the comparison of the objective function and computational time between in in small sized problem Moin, Salhi, and Aziz [1] and this research (with addition of length limit and maximum number of stop) can be seen in Table II and Table III, and an example of small size route visualization in inventory routing problem can be seen Figure 3. The results reported are the results produced from 5 runs.

Based on Table 2, there is no new best solution arise, but the proposed record-to-record travel algorithm can produce the best objective value that are not much different from Moin, Salhi, and Aziz [1] with addition of length limit and maximum number of stop. In addition, the record-to-record travel algorithm can produce shorter computational time when compared to the Moin, Salhi, and Aziz [1]. In this study, delay was used to visualize the change in output, so the CPU time become longer. The average different of result for all comparison from this research to Moin, Salhi, and Aziz [1] is 2.97% and this result is categorized small different. This can be an advantage for choosing the record-to-record travel algorithm for solving the inventory routing problem, because of another approach are not guaranteed to produce the optimal solution and the computational time used are slower.

TABLE II. THE RESULT OF IRP MULTI-PERIOD (12 SUPPLIER)

Data Sets	Record to record		Moin et al. (2011) (HGA1)	
	S12T5	S12T10	S12T5	S12T10
Best Objective	2162.13	4428.78	2099.31	4333.27
Average	2299.86	4531.77	2122.50	4358.07
Std. Dev	95.78	150.18	17.14	20.41
Num. of Vehicle	14	28	14	28
CPU time (s)	34.00	102.00	46.66	111.23

TABLE III. THE RESULT OF IRP MULTI-PERIOD (20 SUPPLIER)

Data Sets	Record to record		Moin et al. (2011) (HGA1)	
	S20T5	S20T10	S20T5	S20T10
Best Objective	3221.20	6842.87	3143.39	6543.08
Average	3491.66	6990.22	3238.61	6674.98
Std. Dev	32.11	52.18	64.56	60.46
Num. of Vehicle	21	44	21	44
CPU time (s)	33.00	59.00	31.81	65.81

VI. CONCLUSION AND FUTURE WORK

In this research, the record-to-record travel algorithm is used to tackle the multi-period IRP and produced good results comparing with previous research. The computational time consumed also reasonable. For future research, the results can be improved by using more powerful local searches such as the two-opt between route or OR-opt in uphill move or downhill move. The number of I, K, and M used can be tested further in order to obtain better results. In addition, the experiment should be made in large quantities and combination to get the correct parameter value.

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